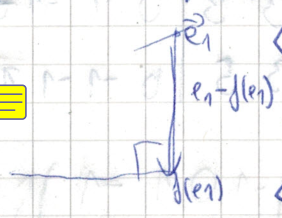
DOMÁCA ÚLOHA (D) – RÓBERTA JURÍKOVÁ

Nájdite maticu ortog. projekcie na podpriestor $S \subseteq \mathbb{R}^4$

$$\vec e_1 = \alpha_1 \vec u_1 + \alpha_2 \vec u_2 + \alpha_3 \vec u_3$$
$$\vec e_2 = \beta_1 \vec u_1 + \beta_2 \vec u_2 + \beta_3 \vec u_3$$
$$\vec e_3 = \gamma_1 \vec u_1 + \gamma_2 \vec u_2 + \gamma_3 \vec u_3$$
$$\vec e_4 = \delta_1 \vec u_1 + \delta_2 \vec u_2 + \delta_3 \vec u_3$$




$$\langle \vec e_1 - f(\vec e_1), \vec u_1\rangle = 0 \Rightarrow \langle f(\vec e_i), \vec u_1\rangle = \langle \vec e_i, \vec u_1\rangle$$
$$\langle \vec e_1 - f(\vec e_1), \vec u_2\rangle = 0 \Rightarrow \langle f(\vec e_i), \vec u_2\rangle = \langle \vec e_i, \vec u_2\rangle$$
$$\langle \vec e_1 - f(\vec e_1), \vec u_3\rangle = 0 \Rightarrow \langle f(\vec e_1), \vec u_3\rangle = \langle \vec e_i, \vec u_3\rangle$$

$$S = [\underbrace{(1,-1,2,0)}_{\vec u_1}, \underbrace{(0,1,-1,-1)}_{\vec u_2}, \underbrace{(0,1,-3,1)}_{\vec u_3}]$$

Ak $i=1$: nepoznáme: $\alpha_1, \alpha_2, \alpha_3$

$$\langle f(\vec e_1), \vec u_1\rangle = \langle \vec e_1, \vec u_1\rangle$$
$$\langle \alpha_1 \vec u_1 + \alpha_2 \vec u_2 + \alpha_3 \vec u_3, \vec u_1\rangle = \alpha_1\langle \vec u_1, \vec u_1\rangle + \alpha_2\langle \vec u_2, \vec u_1\rangle + \alpha_3\langle \vec u_3, \vec u_1\rangle = \langle e_1, u_1\rangle$$
$$\langle f(\vec e_1), \vec u_2\rangle = \alpha_1\langle \vec u_1, \vec u_2\rangle + \alpha_2\langle \vec u_2, \vec u_2\rangle + \alpha_3\langle \vec u_3, \vec u_2\rangle = \langle \vec e_1, \vec u_2\rangle$$
$$\langle f(\vec e_1), \vec u_3\rangle = \alpha_1\langle \vec u_1, \vec u_3\rangle + \alpha_2\langle \vec u_2, \vec u_3\rangle + \alpha_3\langle \vec u_3, \vec u_3\rangle = \langle \vec e_1, \vec u_3\rangle$$

Rovnako tak aj pre $i = 2, 3, 4$. Ľavé strany sa nemenia, menia sa iba neznáme a pravé strany, preto budeme riešiť 4 lineárne systémy naraz.

$$\begin{array}{c} \left(\begin{array}{ccc|cccc} \langle \vec u_1, \vec u_1\rangle & \langle \vec u_2, \vec u_1\rangle & \langle \vec u_3, \vec u_1\rangle & \langle \vec e_1, \vec u_1\rangle & \langle \vec e_2, \vec u_1\rangle & \langle \vec e_3, \vec u_1\rangle & \langle \vec e_4, \vec u_1\rangle \\ \langle \vec u_1, \vec u_2\rangle & \langle \vec u_2, \vec u_2\rangle & \langle \vec u_2, \vec u_3\rangle & \langle \vec e_1, \vec u_2\rangle & \langle \vec e_2, \vec u_2\rangle & \langle \vec e_3, \vec u_2\rangle & \langle \vec e_4, \vec u_2\rangle \\ \langle \vec u_1, \vec u_3\rangle & \langle \vec u_2, \vec u_3\rangle & \langle \vec u_3, \vec u_3\rangle & \langle \vec e_1, \vec u_3\rangle & \langle \vec e_2, \vec u_3\rangle & \langle \vec e_3, \vec u_3\rangle & \langle \vec e_4, \vec u_3\rangle \end{array}\right) \\ \begin{array}{ccccccc} \quad & \quad & \quad & \uparrow \alpha_i & \uparrow \beta_i & \uparrow \gamma_i & \uparrow \delta_i \end{array} \end{array}$$

$$\left(\begin{array}{ccc|cccc} 6 & -3 & -7 & 1 & -1 & 2 & 0 \\ -3 & 3 & +3 & 0 & 1 & -1 & 0 \\ -7 & +3 & 11 & 0 & 1 & -3 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 6 & -3 & -7 & 1 & -1 & 2 & 0 \\ -3 & 3 & -3 & 0 & 1 & -1 & 0 \\ -1 & -6 & 4 & 1 & 0 & -1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|cccc} 1 & 6 & -4 & -1 & 0 & 1 & -1 \\ \cancel{0} & \cancel{3} & \cancel{-13} & \cancel{1} & \cancel{1} & \cancel{0} & \cancel{0} \\ -3 & 3 & -3 & 0 & 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 1 & 6 & -4 & -1 & 0 & 1 & -1 \\ 0 & 1 & -13 & 1 & 1 & 0 & 0 \\ 0 & 21 & -15 & -3 & 1 & 2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 1 & 6 & -4 & -1 & 0 & 1 & -1 \\ 0 & 1 & -13 & 1 & 1 & 0 & 0 \\ 0 & 7 & -5 & -1 & \frac{1}{3} & \frac{2}{3} & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|cccc} 1 & 6 & -4 & -1 & 0 & 1 & -1 \\ 0 & 1 & -13 & 1 & 1 & 0 & 0 \\ 0 & 0 & 86 & & & & \end{array}\right)$$

$$\left(\begin{array}{ccc|cccc} 6 & -3 & -7 & 1 & -1 & 2 & 0 \\ -3 & 3 & 3 & 0 & 1 & -1 & -1 \\ -7 & 3 & 11 & 0 & 1 & -3 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 0 & 3 & -1 & 1 & 1 & 0 & 2 \\ -3 & 3 & 3 & 0 & 1 & -1 & -1 \\ -1 & -3 & 5 & 0 & -1 & -1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 1 & 3 & -5 & 0 & 1 & 1 & -3 \\ 0 & 3 & -1 & 1 & 1 & 0 & -2 \\ 0 & 12 & -12 & 0 & 4 & 2 & -10 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|cccc} 1 & 3 & -5 & 0 & 1 & 1 & -3 \\ 0 & 3 & -1 & 1 & 1 & 0 & -2 \\ 0 & 0 & -8 & -4 & 0 & 2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 1 & 0 & -4 & -1 & 0 & 1 & -1 \\ 0 & 3 & -1 & 1 & 1 & 0 & -2 \\ 0 & 0 & -8 & -4 & 0 & 2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & -8 & -4 & 0 & 2 & -2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & +\frac{1}{2} & 0 & -\frac{1}{4} & \frac{1}{4} \end{array}\right) \sim \left(\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{3} & -\frac{1}{12} & -\frac{7}{12} \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{4} & \frac{1}{4} \end{array}\right)$$

$\Rightarrow \alpha_1 = 1 \quad \alpha_2 = \frac{1}{2} \quad \alpha_3 = \frac{1}{2}$ $\qquad \Rightarrow \vec{e}_1 = (1,-1,2,0) + \left(0,\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right) + \left(0,\frac{1}{2},-\frac{3}{2},\frac{1}{2}\right)$

$\beta_1 = 0 \quad \beta_2 = \frac{1}{3} \quad \beta_3 = 0$ $\qquad \vec{e}_2 = \left(0,\frac{1}{3},-\frac{1}{3},-\frac{1}{3}\right)$

$\gamma_1 = 0 \quad \gamma_2 = -\frac{1}{12} \quad \gamma_3 = -\frac{1}{4}$ $\qquad \vec{e}_3 = \left(0,-\frac{1}{12},\frac{1}{12},\frac{1}{12}\right) + \left(0,-\frac{1}{4},\frac{3}{4},-\frac{1}{4}\right)$

$\delta_1 = 0 \quad \delta_2 = -\frac{7}{12} \quad \delta_3 = \frac{1}{4}$ $\qquad \vec{e}_4 = \left(0,-\frac{7}{12},\frac{7}{12},\frac{7}{12}\right) + \left(0,\frac{1}{4},-\frac{3}{4},\frac{1}{4}\right)$

$$\left.\begin{aligned} \Rightarrow \vec{e}_1 &= (1,0,0,0) \\ \vec{e}_2 &= \left(0,\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3}\right) \\ \vec{e}_3 &= \left(0,-\tfrac{1}{3},\tfrac{5}{6},-\tfrac{1}{6}\right) \\ \vec{e}_4 &= \left(0,-\tfrac{1}{3},-\tfrac{1}{6},\tfrac{5}{6}\right) \end{aligned}\right\} \Rightarrow M_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{5}{6} & -\frac{1}{6} \\ 0 & -\frac{1}{3} & -\frac{1}{6} & \frac{5}{6} \end{pmatrix} =$$

$$= \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & \frac{5}{2} & -\frac{1}{2} \\ 0 & -1 & -\frac{1}{2} & \frac{5}{2} \end{pmatrix}$$