ORTOG. PODOBNOSŤ

$$PAP^T = D$$
$$PAP^{-1} = D$$

A - reálna sym.
P - ortog. $\quad P^T = P^{-1}$, $\quad PP^T = I$

$$P = \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{pmatrix}$$ ← vl. vektory + ortonormálne

- Matica P je ortogonálna $\Leftrightarrow PP^T = I$, t.j. $P^T = P^{-1}$. To platí práve vtedy, keď riadky (stĺpce) matice P tvoria $\boxed{\text{ortonormálnu bázu.}}$
- Pre ľubovoľnú reálnu symetrickú maticu A existujú ortogonálna matica P a diagonálna matica D tak, že
$$PAP^T = PAP^{-1} = D.$$
- Ak A je reálna symetrická matica, tak vlastné vektory zodpovedajúce *rôznym* vlastným hodnotám sú na seba kolmé.
- Ak pracujeme so symetrickou maticou tak riadkové a stĺpcové vlastné vektory sú rovnaké; $\vec{x}A = \lambda\vec{x} \Leftrightarrow A\vec{x}^T = \lambda\vec{x}^T$. Pri zostavení sústavy na nájdenie vlastných vektorov teda teraz netreba transponovať; $(A - \lambda I)^T = (A - \lambda I)$.

$$\begin{pmatrix} A \end{pmatrix} \to \text{vl. č.} = \text{korene } \chi_A(\lambda)$$

$$D = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix} \qquad P = \begin{pmatrix} \vec{r_1} \\ \vec{r_2} \\ \vec{r_3} \\ \vec{r_4} \end{pmatrix}$$

$$PAP^{-1} = D$$

$$\lambda \to \vec{x}(A - \lambda I) = \vec{0}$$
$$(A - \lambda I)^T \vec{x}^T = \vec{0}^T$$

$$\begin{pmatrix} A = PDP^{-1} \\ P^{-1}AP = D \end{pmatrix}$$

$$A = A^T \Rightarrow A - \lambda I = (A - \lambda I)^T$$

1

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

$\chi_A(\lambda) = \lambda\,(\lambda-2)(\lambda-3)$

$0 \quad [\,(2,-1,1)\,] \;\rightarrow\; \frac{1}{\sqrt{6}}(2,-1,1)$

$2 \quad [\,(0,1,1)\,] \;\rightarrow\; \frac{1}{\sqrt{2}}(0,1,1)$

$3 \quad [\,(1,1,-1)\,] \;\rightarrow\; \frac{1}{\sqrt{3}}(1,1,-1)$

$$P = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix} \qquad D = \begin{pmatrix} 0 & & \\ & 2 & \\ & & 3 \end{pmatrix}$$

$$PAP^T = D \iff A = P^T D P$$

$$A = \begin{pmatrix} 0 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{pmatrix} \qquad (0,1,1)A = (0,4,4) \quad 4$$

$$\chi_A(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda & -2 & 2 \\ -2 & \lambda-3 & -1 \\ 2 & -1 & \lambda-3 \end{vmatrix} = \begin{vmatrix} \lambda & -2 & 2 \\ -2 & \lambda-3 & -1 \\ 0 & \lambda-4 & \lambda-4 \end{vmatrix} =$$

$$= (\lambda-4)\begin{vmatrix} \lambda & -2 & 2 \\ -2 & \lambda-3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = (\lambda-4)\begin{vmatrix} \lambda & -4 & 0 \\ -2 & \lambda-2 & 0 \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= (\lambda-4)\begin{vmatrix} \lambda & -4 \\ -2 & \lambda-2 \end{vmatrix} = (\lambda-4)(\lambda^2-2\lambda-8) = $$
$$= (\lambda-4)(\lambda-4)(\lambda+2)$$

$$\chi_A = (\lambda-4)^2(\lambda+2) \qquad 4, 4, -2$$

$$A = \begin{pmatrix} 0 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{pmatrix}$$

4

$$(A-4I) \sim \begin{pmatrix} -4 & 2 & -2 \\ 2 & -1 & 1 \\ -2 & 1 & -4 \end{pmatrix} \sim \begin{pmatrix} \boxed{2 \;\; -1 \;\; 1} \\ 0 \;\; 0 \;\; 0 \\ 0 \;\; 0 \;\; 0 \end{pmatrix}$$

Riešenie $[(1,2,0),(0,1,1)]$

$$\boxed{\begin{matrix} (0,1,1) \\ (1,1,-1) \end{matrix}} \qquad \left(\begin{matrix} 2 & -1 & 1 \\ 0 & 1 & 1 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \end{matrix}\right) \qquad [(1,1,-1)]$$

-2 $\boxed{\text{kolmé na}} \to (2,-1,1)$

$(2,-1,1)A =$

$$(2,-1,1)\begin{pmatrix} 0 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{pmatrix}$$

$= (-4,2,-2) =$
$= -2(2,-1,1)$ ✓

$PAP^T = D$
$P^TDP = A$

$$P = \begin{pmatrix} 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt3} \\ \frac{2}{\sqrt6} & -\frac{1}{\sqrt6} & \frac{1}{\sqrt6} \end{pmatrix} \qquad D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$P^TDP = \begin{pmatrix} 0 & \frac{1}{\sqrt3} & \frac{2}{\sqrt6} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt6} \\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt3} & \frac{1}{\sqrt6} \end{pmatrix} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt3} \\ \frac{2}{\sqrt6} & -\frac{1}{\sqrt6} & \frac{1}{\sqrt6} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{1}{\sqrt3} & \frac{2}{\sqrt6} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt6} \\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt3} & \frac{1}{\sqrt6} \end{pmatrix} \begin{pmatrix} 0 & \frac{4}{\sqrt2} & \frac{4}{\sqrt2} \\ \frac{4}{\sqrt3} & \frac{4}{\sqrt3} & -\frac{4}{\sqrt3} \\ -\frac{4}{\sqrt6} & \frac{2}{\sqrt6} & -\frac{2}{\sqrt6} \end{pmatrix} = \begin{pmatrix} 0 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 4 & 3 \\ -1 & 1 & 3 & 4 \end{pmatrix}$$

$$\chi_A(t) = \begin{vmatrix} t-2 & -1 & -1 & 1 \\ -1 & t-2 & 1 & -1 \\ -1 & 1 & t-4 & -3 \\ 1 & -1 & -3 & t-4 \end{vmatrix} = \begin{vmatrix} t-3 & t-3 & 0 & 0 \\ -1 & t-2 & 1 & -1 \\ -1 & 1 & t-4 & -3 \\ 0 & 0 & t-7 & t-7 \end{vmatrix} =$$

$$= (t-3)(t-7) \begin{vmatrix} 1 & 1 & 0 & 0 \\ -1 & t-2 & 1 & -1 \\ -1 & 1 & t-4 & -3 \\ 0 & 0 & 1 & 1 \end{vmatrix} = (t-3)(t-7) \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & t-1 & 2 & 0 \\ 0 & 2 & t-1 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$= (t-3)(t-7) \begin{vmatrix} t-1 & 2 \\ 2 & t-1 \end{vmatrix} = (t-3)(t-7)\left[(t-1)^2 - 2^2\right] =$$

$$= (t-3)(t-7)(t-3)(t+1) = (t+1)(t-3)^2(t-7)$$

vl. č. : $-1, 3, 3, 7$

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 4 & 3 \\ -1 & 1 & 3 & 4 \end{pmatrix} \qquad -1 \qquad \underset{A+I}{(A+I)^T} \sim \begin{pmatrix} 3 & 1 & 1 & -1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 5 & 3 \\ -1 & 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 4 & 4 & 0 & 0 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 5 & 3 \\ 0 & 0 & 8 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & -2 & 5 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$[(1,-1,-1,1)]$

-1 $[(1,-1,-1,1)]$
$+7$ $[(0,0,1,1)]$
3 $(1,1,0,0)$
3 $(1,-1,1,-1)$

$+7$

$$A-7I=\begin{pmatrix}-5&1&1&-1\\1&-5&-1&1\\1&-1&-3&3\\-1&1&3&-3\end{pmatrix}\sim\begin{pmatrix}-4&-4&0&0\\1&-5&-1&1\\1&-1&-3&3\\0&0&0&0\end{pmatrix}\sim$$

$$A=\begin{pmatrix}2&1&1&-1\\1&2&-1&1\\1&-1&4&3\\-1&1&3&4\end{pmatrix}$$

$$\sim\begin{pmatrix}1&1&0&0\\0&-6&-1&1\\0&-2&-3&3\\0&0&0&0\end{pmatrix}\sim\begin{pmatrix}1&1&0&0\\0&0&8&-8\\0&-2&-3&3\\0&0&0&0\end{pmatrix}\sim\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&-1\\0&0&0&0\end{pmatrix}$$

$[(0,0,1,1)]$

3 $<$ $A-3I$ / kolmé $[(1,-1,-1,1)]$ $[(0,0,1,1)]$

$$\begin{pmatrix}1&-1&-1&1\\0&0&1&1\end{pmatrix}\sim\begin{pmatrix}1&-1&0&2\\0&0&1&1\end{pmatrix}$$

$\cancel{[(1,1,0,0),(2,0,1,-1)]}$

$=[(1,1,0,0),(1,-1,1,-1)]$ ← ortog.

$$\left(\begin{array}{cccc|c}1&-1&0&2&0\\0&0&1&1&0\\1&1&0&0&0\end{array}\right)\sim\begin{pmatrix}1&1&0&0\\0&-2&0&2\\0&0&1&1\end{pmatrix}\sim\begin{pmatrix}1&0&0&1\\0&1&0&-1\\0&0&1&1\end{pmatrix}$$

$[(1,-1,1,-1)]$

-1 $[(1,-1,-1,1)]$
$+7$ $[(0,0,1,1)]$
3 $(1,1,0,0)$
3 $(1,-1,1,-1)$

$$D=\begin{pmatrix}-1&&&\\&3&&\\&&3&\\&&&7\end{pmatrix}\qquad PAP^T=D$$

$$P=\begin{pmatrix}\frac12&-\frac12&-\frac12&\frac12\\\frac12&-\frac12&\frac12&-\frac12\\\frac1{\sqrt2}&\frac1{\sqrt2}&0&0\\0&0&\frac1{\sqrt2}&\frac1{\sqrt2}\end{pmatrix}\qquad P^TDP=A$$

$$P^TDP = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{3}{2} & \frac{3}{2} & -\frac{3}{2} \\ \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{7}{\sqrt{2}} & \frac{7}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 4 & 3 \\ -1 & 1 & 3 & 4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

$\chi_A(t) = t^2(t-4)^2$ vl. č. : 0, 0, 4, 4

k 0:

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$[(1,-1,0,0), (2,0,1,1)] = [(1,-1,0,0), (1,1,1,1)]$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right)$$

0 $[(1,-1,0,0),(1,1,1,1)]$

4 $[(0,0,1,-1),(1,1,-1,-1)]$

$A-4I$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right)$$

$(0,0,1,-1)$
$(1,1,-1,-1)$

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$PAP^T = D \iff P^T D P = A$

$$P^T DP = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & -2 & -2 \\ 0 & 0 & 2\sqrt{2} & -2\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

$PAP^T \stackrel{?}{=} D$

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ -1 & 1 & 1 & 2 \end{pmatrix}$$

$$\chi_A(\lambda) = \begin{vmatrix} \lambda-2 & -1 & -1 & 1 \\ -1 & \lambda-2 & 1 & -1 \\ -1 & 1 & \lambda-2 & -1 \\ 1 & -1 & -1 & \lambda-2 \end{vmatrix} = \begin{vmatrix} \lambda-3 & \lambda-3 & 0 & 0 \\ -1 & \lambda-2 & 1 & -1 \\ -1 & 1 & \lambda-2 & -1 \\ 0 & 0 & \lambda-3 & \lambda-3 \end{vmatrix} =$$

$$= (\lambda-3)^2 \begin{vmatrix} 1 & 1 & 0 & 0 \\ -1 & \lambda-2 & 1 & -1 \\ -1 & 1 & \lambda-2 & -1 \\ 0 & 0 & 1 & 1 \end{vmatrix} = (\lambda-3)^2 \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & \lambda-1 & 2 & 0 \\ 0 & 2 & \lambda-1 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$= (\lambda-3)^2 \begin{vmatrix} \lambda-1 & 2 \\ 2 & \lambda-1 \end{vmatrix} = (\lambda-3)^2 (\lambda-3)(\lambda+1) = (\lambda-3)^3(\lambda+1)$$

$$(\lambda-1)^2 - 2^2 = (\lambda-3)(\lambda+1) \qquad 3,3,3,\ -1$$

3

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ -1 & 1 & 1 & 2 \end{pmatrix} \qquad A - 3I = \begin{pmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Risolvent: ~~[(1,1,0,0), (1,0,1,0), (1,0,0,-1)]~~

$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \longrightarrow \boxed{\begin{matrix} (1,1,0,0) \\ (0,0,1,1) \\ (1,-1,1,-1) \end{matrix}}$$

3 $\qquad$ -1 $\quad (1,-1,-1,1)$

$$D = \begin{pmatrix} 3 & & & \\ & 3 & & \\ & & 3 & \\ & & & -1 \end{pmatrix} \qquad P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$P = \frac{1}{2}\boxed{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

± 1, kolmé riadky

Hadamardova matica $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$PAP^T = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ -1 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & -3 & -3 \\ 3 & -3 & 3 & -3 \\ -1 & 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 12 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$)P = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & -3 & -3 \\ 3 & -3 & 3 & -3 \\ -1 & 1 & 1 & -1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 8 & 4 & 4 & -4 \\ 4 & 8 & -4 & 4 \\ 4 & -4 & 8 & 4 \\ -4 & 4 & 4 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ -1 & 1 & 1 & 2 \end{pmatrix}$$