

Fibonacciho postupnosť

25. apríla 2021

Fibonacciho postupnost

$$F_{n+1} = F_n + F_{n-1} \quad (1)$$

$$\begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} \quad (2)$$

Fibonacciho postupnosť

$$x^2 - x - 1 = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{5}}{2}.$$

Na základe Viétových vzťahov pre ne platí

$$\lambda_1 + \lambda_2 = 1,$$

$$\lambda_1 \lambda_2 = -1.$$

Fibonacciho postupnost

$$F_n = \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}, \quad (3)$$

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}. \quad (4)$$

Fibonacciho posturnost

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$$

Fibonacciho postupnost

$$\sum_{k=1}^n F_k = F_{n+2} - 1$$

$$F_{n+1}F_{n-1} - F_n^2 = (-1)^n$$

$$F_{m+n} = F_m F_{n+1} + F_{m-1} F_n$$

$$F_{2n} = F_n(F_{n+1} + F_{n-1})$$

$$F_{2n+1} = F_{n+1}^2 + F_n^2$$