

# Lineárne rekurencie

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# Sústava rekurencií

$$x_{n+1} = x_n + 2y_n$$

$$y_{n+1} = 2x_n + y_n$$

## Sústava rekurencií

$$x_{n+1} = x_n + 2y_n$$

$$y_{n+1} = 2x_n + y_n$$

$$x_{n+1} + y_{n+1} = 3(x_n + y_n)$$

$$x_{n+1} - y_{n+1} = -(x_n - y_n)$$

## Lineárne rekurencie

$$A_{n+1} = aA_n + bA_{n-1}, \quad (1)$$

$$\begin{pmatrix} A_{n+1} \\ A_n \end{pmatrix} = \begin{pmatrix} a & b \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A_n \\ A_{n-1} \end{pmatrix} \quad (2)$$

## Lineárne rekurencie

$$A = \begin{pmatrix} a & b \\ 1 & 0 \end{pmatrix}$$

$$\text{ch}_A(x) = \begin{vmatrix} a-x & b \\ 1 & -x \end{vmatrix} = x(x-a) - b = x^2 - ax - b.$$

$$\lambda_1 + \lambda_2 = a$$

$$\lambda_1 \cdot \lambda_2 = -b$$

(3)

## Lineárne rekurencie

$$\begin{pmatrix} A_{n+1} \\ A_n \end{pmatrix} = A \begin{pmatrix} A_n \\ A_{n-1} \end{pmatrix} = A^2 \begin{pmatrix} A_{n-1} \\ A_{n-2} \end{pmatrix} = \dots = A^n \begin{pmatrix} A_1 \\ A_0 \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} A_{n+1} \\ A_n \end{pmatrix} = A^n \begin{pmatrix} A_1 \\ A_0 \end{pmatrix} = P^{-1} J^n P \begin{pmatrix} A_1 \\ A_0 \end{pmatrix}$$

## Lineárne rekurencie

$$J^n = \begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix}$$

$$A_n = c_1 \lambda_1^n + c_2 \lambda_2^n$$

$$J^n = \begin{pmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{pmatrix}$$

$$A_n = c_1 \lambda^n + c_2 n \lambda^{n-1}$$

# Lineárne rekurencie

$$A_{n+k} = c_{k-1}A_{n+k-1} + c_{k-2}A_{n+k-2} + \dots + c_1A_{n+1} + c_0A_n$$

$$x^k - c_{k-1}x^{k-1} - c_{k-2}x^{k-2} + \dots - c_1x - c_0 = 0$$



## Fibonacciho postupnosť

$$F_{n+1} = F_n + F_{n-1} \quad (5)$$

$$\begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} \quad (6)$$

## Fibonacciho postupnosť

$$x^2 - x - 1 = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{5}}{2}.$$

$$\lambda_1 + \lambda_2 = 1,$$

$$\lambda_1 \lambda_2 = -1.$$

## Fibonacciho postupnosť

$$F_n = \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}, \quad (7)$$

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}. \quad (8)$$

## Fibonacciho postupnosť

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$$

## Fibonacciho postupnosť

$$\sum_{k=1}^n F_k = F_{n+2} - 1$$

$$F_{n+1}F_{n-1} - F_n^2 = (-1)^n$$

$$F_{m+n} = F_m F_{n+1} + F_{m-1} F_n$$

$$F_{2n} = F_n(F_{n+1} + F_{n-1})$$

$$F_{2n+1} = F_{n+1}^2 + F_n^2$$