DETERMINANTY

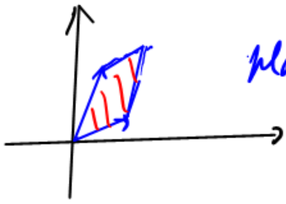
Pre ŠTVORCOVÉ matice $A \in M_{n,n}(F)$

OZN: $|A| = \det(A) \in F$

$$\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

MOTIVÁCIA - GEOMETRIA: objem/plocha (+ znamienko)

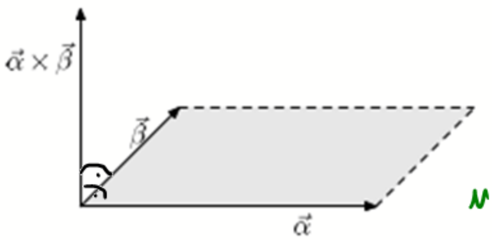
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad \vec{\alpha} = (a_{11}, a_{12}) \quad \vec{\beta} = (a_{21}, a_{22})$$



plocha rovnobežníka = ?

VEKTOROVÝ SÚČIN:
dĺžka = plocha

$$\vec{\alpha} = (a_{11}, a_{12}, 0)$$
$$\vec{\beta} = (a_{21}, a_{22}, 0)$$
$$\vec{\alpha} \times \vec{\beta} = (0, 0, a_{11}a_{22} - a_{12}a_{21})$$



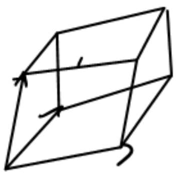
$$S = |a_{11}a_{22} - a_{12}a_{21}|$$

NESKÔR

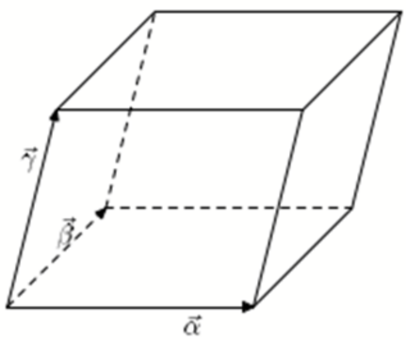
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

determinant

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a & \cdots & a_{33} \end{pmatrix}$$

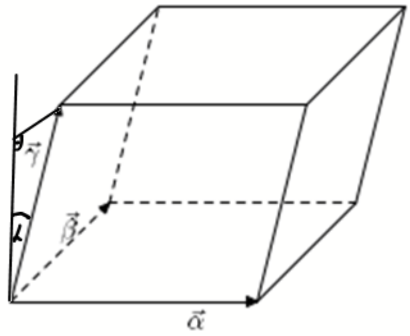


$\vec{\alpha} = (a_{11}, a_{12}, a_{13})$
$\vec{\beta} = (a_{21}, a_{22}, a_{23})$
$\vec{\gamma} = (a_{31}, a_{32}, a_{33})$



V = ?
objem.

Obr. 6.2: Rovnobežnosten určený 3 vektormi v $\mathbb{R}^3$



Obr. 6.2: Rovnobežnosten určený 3 vektormi v $\mathbb{R}^3$

$$V = S \cdot v$$
$$v = |\vec{\gamma}| \cdot \cos\alpha$$
$$S = |\vec{\alpha} \times \vec{\beta}|$$
$$\pm V = |\vec{\alpha} \times \vec{\beta}| \cdot |\vec{\gamma}| \cos\alpha = \underbrace{(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\gamma}}_{\text{skal. súč.}}$$

$$\left.\begin{array}{l}\vec{\alpha} = (a_{11}, a_{12}, a_{13}) \\ \vec{\beta} = (a_{21}, a_{22}, a_{23}) \\ \vec{\gamma} = (a_{31}, a_{32}, a_{33})\end{array}\right\} \quad \begin{array}{l}\vec{\alpha} \times \vec{\beta} = (a_{12}a_{23} - a_{13}a_{22},\ a_{13}a_{21} - a_{11}a_{23},\ a_{11}a_{22} - a_{12}a_{21}) \\ \vec{\gamma} = (\quad a_{31} \quad , \quad a_{32} \quad , \quad a_{33} \quad)\end{array}$$

$$(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\gamma} = a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} + a_{11}a_{22}a_{33} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

↑
GEOM.
FORMÁLNA DEF.
↓

6.2 Definícia determinantu

Definícia 6.2.1. V tejto kapitole budeme označovať ako S_n množinu všetkých permutácií množiny $\{1, 2, \ldots, n\}$.

Dvojica $(\varphi(k), \varphi(s))$ sa volá *inverzia* permutácie φ, ak $k < s$ ale $\varphi(k) > \varphi(s)$. Počet inverzií permutácie φ budeme označovať $i(\varphi)$.

$$\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \qquad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

2 1
4 3

$i(\varphi) = 2$

4 3
4 2
4 1
3 2
3 1
2 1

Definícia 6.2.3. Nech A je matica typu $n \times n$ nad poľom F, $A = ||a_{ij}||$. *Determinant matice* A je

$$|A| = \sum_{\varphi \in S_n} (-1)^{i(\varphi)} a_{1\varphi(1)} a_{2\varphi(2)} \dots a_{n\varphi(n)}. \tag{6.1}$$

Symbolom $\sum\limits_{\varphi \in S_n}$ rozumieme, že sčitujeme cez celú množinu S_n, teda pre každú permutáciu $\varphi \in S_n$ pripočítame jeden sčítanec uvedeného tvaru. (Množina S_n je konečná, teda takýto súčet je jednoznačne definovaný.)

$(-1)^{i(\varphi)} a_{1\varphi(1)} a_{2\varphi(2)} \dots a_{n\varphi(n)}$ ← súčin: 1 prvok z každého r.

$n = 1$ $\quad |A| = |a_{11}| = a_{11}$ $\qquad (-1)^{i(\varphi)} a_{1\varphi(1)} a_{2\varphi(2)}$

$n = 2$: $\quad \varphi = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$ $\quad i(\varphi) = 0$ $\qquad + \; a_{11} a_{22}$

$\varphi = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ $\quad i(\varphi) = 1$ $\qquad - \; a_{12} a_{21}$

$$\boxed{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.}$$

$$|A| = \sum_{\varphi \in S_n} (-1)^{i(\varphi)} a_{1\varphi(1)} a_{2\varphi(2)} \dots a_{n\varphi(n)}.$$

S_3 .. perm. $\{1, 2, 3\}$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

φ	$i(\varphi)$	inverzie
$\binom{1\,2\,3}{1\,2\,3}$	0	
$\binom{1\,2\,3}{1\,3\,2}$	1	(3, 2)
$\binom{1\,2\,3}{2\,1\,3}$	1	(2, 1)
$\binom{1\,2\,3}{2\,3\,1}$	2	(2, 1) (3, 1)
$\binom{1\,2\,3}{3\,1\,2}$	2	(3, 1) (3, 2)
$\binom{1\,2\,3}{3\,2\,1}$	3	(3, 2) (3, 1) (2, 1)

$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$

$- a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$

$$\left[(\vec{I} \times \vec{B}) \cdot \vec{J} = a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} + a_{11}a_{22}a_{33} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} \right]$$

PRE 3x3:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$$

$a_{11}a_{22}a_{33} + \underline{a_{21}a_{32}a_{13}} + a_{31}a_{12}a_{23}$

$- a_{31}a_{22}a_{13} - a_{11}a_{32}a_{23} - a_{21}a_{12}a_{33}$

Sarrusovo pravidlo

Pre 4x4 → 4! sčítancov → BUDEME RÁTAŤ INAK → Laplaceov rozvoj / pomocou ERO

↳ permutácií