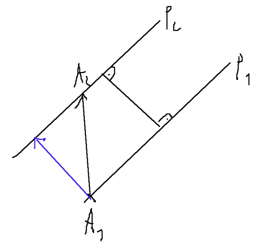
1.1. [P, 1376] Ukážte, že vzdialenosť medzi dvoma afinnými podpriestormi $P_1 = A_1 + V_1$ a $P_2 = A_2 + V_2$ sa rovná dĺžke ortogonálnej projekcie vektora $\overrightarrow{A_1A_2}$ do priestoru $V^\perp$, kde $V = V_1 + V_2$.

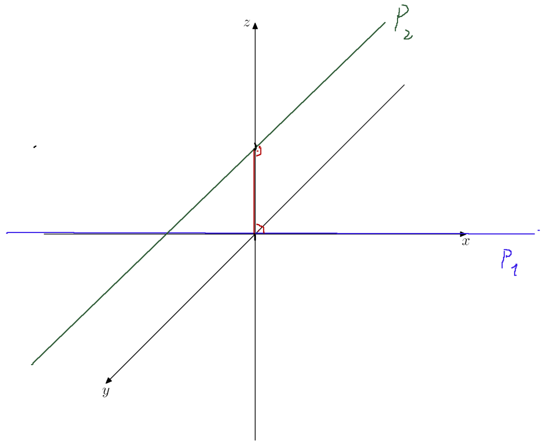
1.2. Ukážte, že vzdialenosť medzi dvoma afinnými priestormi $P_1 = A_1 + V_1$ a $P_2 = A_2 + V_2$ sa rovná vzdialenosti bodu A_1 od afinného podpriestoru $A_2 + V$, kde $V = V_1 + V_2$.

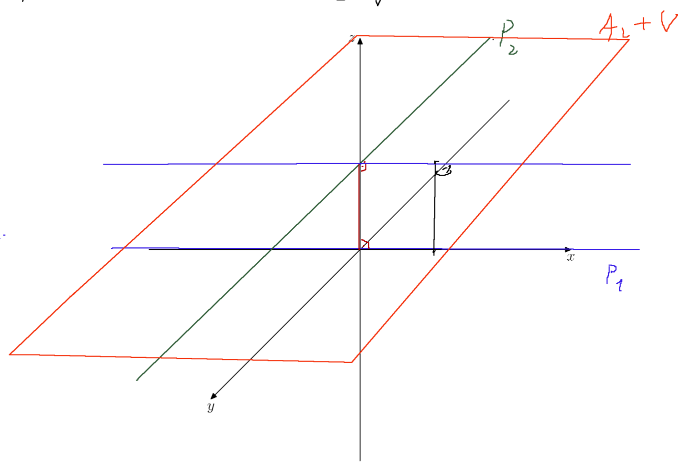
1.1



1.2. Ukážte, že vzdialenosť medzi dvoma afinnými priestormi $P_1 = A_1 + V_1$ a $P_2 = A_2 + V_2$ sa rovná vzdialenosti bodu A_1 od afinného podpriestoru $A_2 + V$, kde $V = V_1 + V_2$.

= vzdialenosť $A_1 + V$ a $A_2 + V$





1.2. Ukážte, že vzdialenosť medzi dvoma afinnými priestormi $P_1 = A_1 + V_1$ a $P_2 = A_2 + V_2$ sa rovná vzdialenosti bodu A_1 od afinného podpriestoru $A_2 + V$, kde $V = V_1 + V_2$.

$$d(P_1, P_2) = \inf\{d(x,y);\ x \in P_1, y \in P_2\}$$

⊗ rovnobežné rovnaké dimenzie

$$d(A_1, A_2+V) = \inf\{d(A_1, y);\ y \in A_2+V\}$$

$$\parallel ⊗$$

$$d(A_1+V, A_2+V)$$

$$d(P_1, P_2) \geq d(A_1+V, A_2+V) \qquad P_1 \subseteq A_1+V, \quad P_2 \subseteq A_2+V$$

$$\{d(x,y);\ x\in P_1, y\in P_2\} \subseteq \{d(x,y);\ x\in A_1+V, y\in A_2+V\}$$

$$\inf\{d(x,y);\ x\in P_1, y\in P_2\} \geq \inf\{d(x,y);\ x\in A_1+V, y\in A_2+V\}$$

$$A \subseteq B \Rightarrow \inf A \geq \inf B$$
$$\sup A \leq \sup B$$

$$d(P_1, P_2) \leq d(A_1, A_2+V)$$

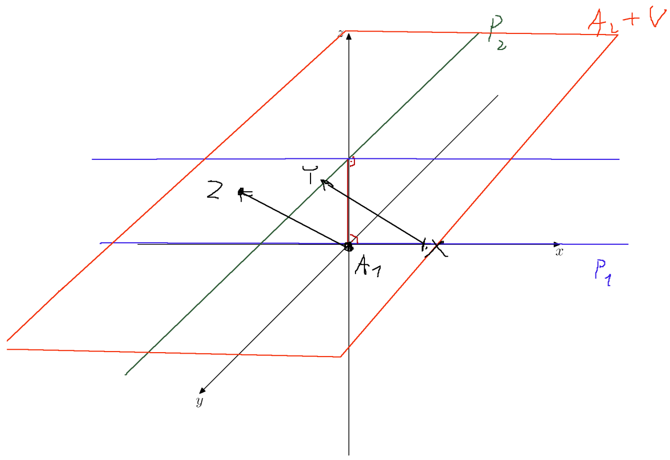
$$\Downarrow$$

$$d(X,Y);\ X\in P_1, Y\in P_2$$

$$d(A_1, Z) \qquad Z \in A_2+V$$

$$Z = A_1 + \overrightarrow{XY} \in A_2+V$$

$$= A_2 + \underbrace{\boxed{\overrightarrow{A_2A_1} + \overrightarrow{XY}}}_{\in V} \in A_2+V$$



$$X \in P_1 \qquad X = A_1 + \vec{v_1} \qquad \vec{v_1} \in V_1$$
$$Y \in P_2 \qquad Y = A_2 + \vec{v_2} \qquad \vec{v_2} \in V_2$$

$$\overrightarrow{XY} = Y - X = (A_2 + \vec{v_2}) - (A_1 + \vec{v_1})$$
$$\overrightarrow{XY} = \overrightarrow{A_1A_2} + \vec{v_2} - \vec{v_1}$$
$$\overrightarrow{A_2A_1} + \overrightarrow{XY} = \vec{v_2} - \vec{v_1} \in V = V_1 + V_2$$

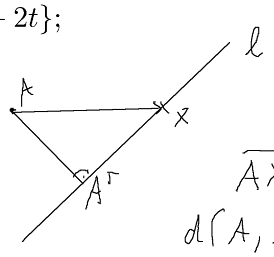
1.3. [BPC, 34.21] Nájdite vzdialenosť bodu A od priamky l:

a) $A = (0,3,2,-5)$, $l = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4; x_1 = 1+t, x_2 = -t, x_3 = 2+2t, x_4 = -2+2t\}$;

$B \in l$ $\quad B = (1,0,2,-2)$

$\vec{u} = (1,-1,2,2)$

$B + t\vec{u}$

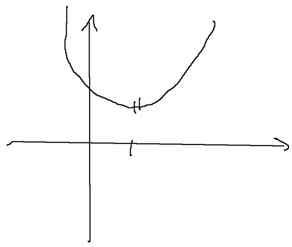


$\overrightarrow{AX} = (1+t, -3-t, 2t, 3+2t)$

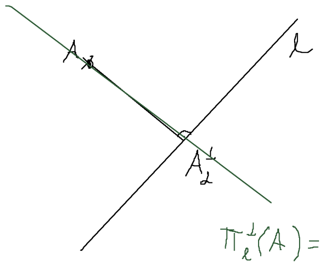
$d(A,l) = \min\{d(A,X);\ X \in l\}$

min $\leftarrow$

$$\begin{aligned}
f(t) = |\overrightarrow{AX}|^2 = \\
&(1+t)^2 + (-3-t)^2 + (2t)^2 + (3+2t)^2 \\
&= (t^2+2t+1) + (t^2+6t+9) + 4t^2 + (4t^2+12t+9) \\
&= 10t^2 + 20t + 19 \\
&= \boxed{10(t+1)^2 + 9}
\end{aligned}$$



min; $\boxed{t=-1}$ $\quad$ rovná $\boxed{9}$ $\quad \rightarrow$ vzd. $\underline{\underline{3}}$



$\pi_l^\perp(A) = l_A^\perp$

$d(\alpha_A^\perp) + d(\alpha) = n$

Kolmopremietací priestor:

nadrovina

$\downarrow$

norm. vektor $\quad \vec{u} = (1,-1,2,2)$

$A = (0,3,2,-5)$

$\boxed{x_1 - x_2 + 2x_3 + 2x_4 = -9}$

$$\begin{aligned}
x_1 - x_2 + 2x_3 + 2x_4 &= -9 \\
(1+t) - (-t) + 2(2+2t) + 2(-2+2t) &= -9 \\
10t + 1 &= -9 \\
10t &= -10 \\
\boxed{t = -1}
\end{aligned}$$

$t = -1 \rightsquigarrow$ $\quad x_1 = 1+t$

$x_2 = -t$

$x_3 = 2+2t$

$x_4 = -2+2t$

$A^\perp = (0,1,0,-4)$

$A = (0,3,2,-5)$

$|AA^\perp| = |\overrightarrow{AA^\perp}| = \sqrt{0^2+2^2+2^2+1^2} = \sqrt{9} = \underline{\underline{3}}$

1.4. [BPC, 34.23] Nájdite vzdialenosť medzi priamkami l_1 a l_2:

a) l_1: $x_1 = 1 + t$, $x_2 = -1$, $x_3 = -t$, $x_4 = -2 + t$; l_2: $x_1 = 4 + t$, $x_2 = 2t$, $x_3 = 1 + t$, $x_4 = t$

b) $l_1 = \{(2 + t, -1 - 2t, 2 + 2t, 1 - t); t \in \mathbb{R}\}$; $l_2 = \{(3 - t, 1 + 2t, -1 - 2t, 2 + t); t \in \mathbb{R}\}$;

c) $l_1 = \{(3 + t, 2, t, 3 + t, -t); t \in \mathbb{R}\}$; $l_2 = \{(1 + 2t, 2t, 1 - t, t, 2); t \in \mathbb{R}\}$;

d) $l_1 = \{(1 + t, 2t, 1 - t, -1 + t, t); t \in \mathbb{R}\}$; $l_2 = \{(3 + t, -2t, -1 - t, 1 + t, 2 + t); t \in \mathbb{R}\}$;

e) $l_1 = \{(1 - 2t, 0, t, 1 + t, 2); t \in \mathbb{R}\}$; $l_2 = \{(-1 + t, -1 + t, 0, 1, -2 - t); t \in \mathbb{R}\}$;

[Výsledky: a) $\sqrt{3}$, b) $\sqrt{5}$; c) 2; d) $2\sqrt{2}$; e) 4]

a)

$$x_1 = 1 + t \qquad x_1 = 4 + s$$
$$x_2 = -1 \qquad x_2 = 2s$$
$$x_3 = -t \qquad x_3 = 1 + s$$
$$x_4 = -2 + t \qquad x_4 = s$$

$f(s, t) \to \min$

$A_1 = (1, -1, 0, -2)$ $\quad$ $A_2 = [4, 0, 1, 0]$ $\quad$ $\boxed{\overrightarrow{A_1A_2} = (3, 1, 1, 2)}$

$\vec{n}_1 = (1, 0, -1, 1)$ $\quad$ $\vec{n}_2 = (1, 2, 1, 1)$

$V = [\vec{n}_1, \vec{n}_2]$

$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 1 & 2 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 2 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$V^\perp$ $\quad (1, -1, 1, 0) = \vec{b}_2$

$(1, 0, 0, -1) = \vec{b}_1$

$\langle \vec{b}_1, \vec{b}_2 \rangle = 1$

$c_{21} = -\frac{\langle \vec{b}_1, \vec{b}_2 \rangle}{\langle \vec{b}_1, \vec{b}_1 \rangle} = -\frac{1}{2}$

$\vec{a}_1 = \vec{b}_1$

$\vec{a}_2 = \vec{b}_2 + c_{21}\vec{b}_1 = (\frac{1}{2}, -1, 1, \frac{1}{2})$

$V^\perp = [(1, 0, 0, -1), (1, -2, 2, 1)]$

ortog. báza $V^\perp$

$$M = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}(1\ 0\ 0\ -1) + \frac{1}{10}\begin{pmatrix} 1 \\ -2 \\ 2 \\ 1 \end{pmatrix}(1\ -2\ 2\ 1) =$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{10}\begin{pmatrix} 1 & -2 & 2 & 1 \\ -2 & 4 & -4 & -2 \\ 2 & -4 & 4 & 2 \\ 1 & -2 & 2 & 1 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 6 & -2 & 2 & -4 \\ -2 & 4 & -4 & -2 \\ 2 & -4 & 4 & 2 \\ -4 & -2 & 2 & 6 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} \ \end{pmatrix}$$

$$\frac{1}{5}(3, 1, 1, 2)\begin{pmatrix} 3 & -1 & 1 & -2 \\ -1 & 2 & -2 & -1 \\ 1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 3 \end{pmatrix} = \frac{1}{5}(5, -5, 5, 0) = \underbrace{(1, -1, 1, 0)}_{\in V^\perp}$$

$\boxed{\overrightarrow{A_1A_2} = (3, 1, 1, 2)}$ $\qquad$ $d(l_1, l_2) = \|(1, -1, 1, 0)\| = \sqrt{3}$

1.10. [S, 1375] Nájsť vzdialenosť medzi priamkou l a rovinou P ak:

a) $l = (9, -2, -1, -1) + [(2, -2, -1, -1)]$; $P = (2, 1, 3, -3) + [(3, -2, 2, 0), (-5, 2, 0, 2)]$;

b) $l = (2, 4, 0, 14) + [(0, 1, -2, 5)]$; $P = (4, 1, -2, 5) + [(-1, 1, -1, 5), (1, 1, -3, 3)]$.

[Výsledky: a) $\frac{27}{5}$; b) $\sqrt{6}$]

$l:\ A = (9, -2, -1, -1) \qquad (2, -2, -1, 1) \to V_1$

$P = (2, 1, 3, -3) \qquad \left.\begin{matrix}(3, -2, 2, 0)\\ (-5, 2, 0, 2)\end{matrix}\right\} V_2$

$V = V_1 + V_2$

$$\begin{pmatrix} 2 & -2 & -1 & -1 \\ 3 & -2 & 2 & 0 \\ -5 & 2 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & -2 & -1 & -1 \\ 3 & -2 & 2 & 0 \\ -2 & 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & -2 & -1 & -1 \\ 1 & 0 & 3 & 1 \\ 1 & 0 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 4 & 2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$d(V_1 + V_2) = 3$

$d(V_1 \cap V_2) = 0$

$$V^{\perp} = [(-1, -\tfrac{1}{2}, 1, -2)] = [(2, 1, -2, 4)]$$

Nadrovina: $P + V$

$$2x_1 + x_2 - 2x_3 + 4x_4 = -13 \quad \Leftarrow P = (2, 1, 3, -3)$$

$$2x_1 + x_2 - 2x_3 + 4x_4 + 13 = 0$$

$A = (9, -2, -1, -1)$

$$d(A, P+V) = \frac{|2a_1 + a_2 - 2a_3 + 4a_4 + 13|}{\sqrt{2^2 + 1^2 + 2^2 + 4^2}} = \frac{|2\cdot 9 - 2 + 2 - 4 + 13|}{\sqrt{25}} = \boxed{\frac{27}{5}}$$

INAK: Projekcia do $V^{\perp} = [(2, 1, -2, 4)]$ $\qquad M = \frac{1}{25}\begin{pmatrix} 2 \\ 1 \\ -2 \\ 4 \end{pmatrix}(2\ 1\ -2\ 4)$

$\overrightarrow{AP} \cdot M$

1.8. $\alpha \equiv \begin{cases} x_1 = 3u + v \\ x_2 = 1 + 2u + v \\ x_3 = -4u - 2v \\ x_4 = 2 + u + v \end{cases}$ $\beta \equiv \begin{cases} x_1 - 2x_2 + x_4 - 1 = 0 \\ 2x_2 + x_3 + 1 = 0 \end{cases}$ $\varrho(\alpha, \beta) = ?$ [Výsledok: $\frac{\sqrt{10}}{2}$]

$A = (0, 1, 0, 2)$

$V_\alpha : \begin{pmatrix} 3 & 2 & -4 & 1 \\ 1 & 1 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & -1 & 2 & -2 \\ 1 & 1 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & -1 & 2 & -2 \\ 1 & 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 2 \end{pmatrix}$

$V_\beta^\perp \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{pmatrix}$ $V_\beta = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 2 \end{pmatrix}$ $\longrightarrow \boxed{V_\alpha = V_\beta}$

$\Downarrow$

$\left(V_\alpha^\perp = V_\beta^\perp \right)$

$\beta_P = \left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 & -1 \end{array}\right)$ $B = (0, 0, -1, 1)$

rovnobežné

$$\boxed{d(\alpha, \beta) = d(A, \beta) = d(\alpha, B)}$$

$$d(\alpha, \beta) = d(A, B + \underbrace{V_\alpha + V_\beta}_{= V_\alpha = V_\beta}) = d(A, B)$$

N.D.Ú. dopočítať