Bertrandov postulát

14. októbra 2020

Lema o d.c.c.

Lema

Pre ľubovoľné $x \in \mathbb{R}$ *platí* $\lfloor 2x \rfloor - 2\lfloor x \rfloor \in \{0, 1\}$. *Presnejšie,*

$$\lfloor 2x \rfloor - 2\lfloor x \rfloor = \begin{cases} 0, & ak\ 0 \leq \{x\} < \frac{1}{2}; \\ 1, & ak\ \frac{1}{2} \leq \{x\}. \end{cases}$$

Lema o d.c.c.

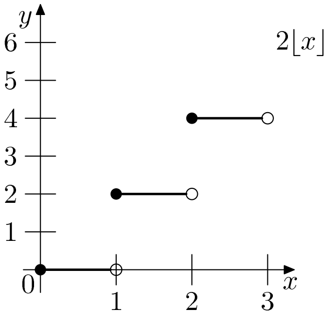

Figure: Funkcie $2\lfloor x \rfloor$ a $\lfloor 2x \rfloor$

Lema o d.c.c.

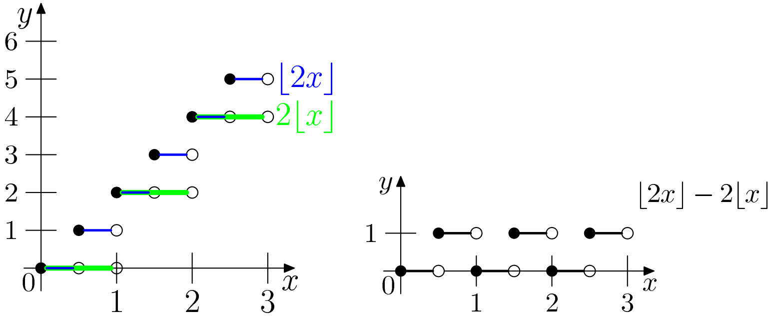

Figure: Funkcie $\lfloor 2x \rfloor$ a $2\lfloor x \rfloor$ a ich rozdiel

Legendrova veta

Veta (Legendre)

Prvočíslo p sa v kanonickom rozklade čísla $n!$ vyskytuje v mocnine rovnej

$$\sum_{k=1}^{\infty} \left\lfloor \frac{n}{p^k} \right\rfloor .$$

Bertrandov postulát

Veta (Bertrandov postulát)

Pre každé $n \in \mathbb{N}$ *existuje prvočíslo* p *také, že*

$$n < p \leq 2n.$$

Dôkaz Bertrandovho postulátu

$$\binom{2n}{n} = \prod_{p \leq n} p^{r(p,n)}. \tag{1}$$

$$r(p,n) = \sum_{j=1}^{\infty} \left(\left\lfloor \frac{2n}{p^j} \right\rfloor - 2 \left\lfloor \frac{n}{p^j} \right\rfloor \right). \tag{2}$$

- $p > \sqrt{2n} \Rightarrow r(p,n) = \lfloor \frac{2n}{p} \rfloor - 2\lfloor \frac{n}{p} \rfloor$
- $p > \frac{2}{3}n \Rightarrow r(p,n) = 0$
- $p^{r(p,n)} \leq 2n$

Horný odhad pre $\binom{2n}{n}$

$\prod_{p \le x} p < 4^x$

$$\prod_{\sqrt{2n} < p \le \frac{2}{3}n} p^{r(p,n)} \le \prod_{p \le \frac{2}{3}n} p < 4^{\frac{2}{3}n}$$

$$\binom{2n}{n} \le (2n)^{\sqrt{2n}} 4^{\frac{2}{3}n}$$

Dolný odhad pre $\binom{2n}{n}$

$$\binom{2n}{n} \geq \frac{4^n}{2n}$$

$$(2n)^{\sqrt{2n}} 4^{\frac{2}{3}n} \geq \frac{4^n}{2n}$$

$$(2n)^{\sqrt{2n}+1} \geq 4^{\frac{n}{3}}$$