Dobre definované operácie

5. októbra 2023

Nerovnosť je dobre definovaná

$$\left.\begin{array}{c} |X| = |X'|, |Y| = |Y'| \\ |X| \leq |Y| \end{array}\right\} \Rightarrow |X'| \leq |Y'|$$

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "h_1"'] & Y \arrow[d, "h_2"] \\ X' \arrow[r, "?"'] & Y' \end{tikzcd}$$

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$$\begin{tikzcd}
X \arrow[r, "f"] \arrow[d, shift right, "h_1"'] & Y \arrow[d, "h_2"] \\
X' \arrow[u, shift right, "h_1^{-1}"'] \arrow[r, "h_2 \circ f \circ h_1^{-1}"'] & Y'
\end{tikzcd}$$

Súčet je dobre definovaný

$$\left.\begin{array}{l} |A| = |A'|, |B| = |B'| \\ A \cap B = A' \cap B' = \emptyset \end{array}\right\} \Rightarrow |A \cup B| \leq |A' \cup B'|$$

Súčin je dobre definovaný

$$|A| = |A'|, |B| = |B'| \Rightarrow |A \times B| = |A' \times B'|$$

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$$|A| = |A'|, |B| = |B'| \Rightarrow |A \times B| = |A' \times B'|$$

Ak $f\colon A \to B$ a $g\colon A' \to B'$ sú bijekcie, tak

$$f \times g\colon A \times A' \to B \times B'$$

je bijekcia.

Mocnina je dobre definovaná

$|A| = |C|$, $|B| = |D| \Rightarrow |A^B| = |C^D|$
Chceme bijekciu $\varphi\colon A^B \to C^D$.

$$\begin{tikzcd} B \arrow[r, "g"] \arrow[d, "h"'] & D \arrow[d, "?"] \\ A \arrow[r, "f"'] & C \end{tikzcd}$$

Mocnina je dobre definovaná

$$|A| = |C|,\ |B| = |D| \Rightarrow |A^B| = |C^D|$$

$$\begin{tikzcd} B \arrow[r, shift right, "g"'] \arrow[d, "h"'] & D \arrow[l, shift right, "g^{-1}"'] \arrow[d, "f \circ h \circ g^{-1}"] \\ A' \arrow[r, "f"'] & C \end{tikzcd}$$

$$\varphi\colon A^B \to C^D$$

$$\varphi(h) = f \circ h \circ g^{-1}$$

Mocnina je dobre definovaná

$\psi\colon C^D \to A^B$ podobným spôsobom ako sme našli φ.

$$\begin{tikzcd} B \arrow[r, "g"] \arrow[d, "?"'] & D \arrow[d, "k"] \\ A \arrow[r, "f"'] & C \end{tikzcd}$$

$$\psi(k) = f^{-1} \circ k \circ g.$$

Mocnina je dobre definovaná

$$\psi(\varphi(h)) = f^{-1} \circ (f \circ h \circ g^{-1}) \circ g = (f^{-1} \circ f) \circ h \circ (g^{-1} \circ g) = h$$
$$\varphi(\psi(k)) = f \circ (f^{-1} \circ k \circ g) \circ g^{-1} = (f \circ f^{-1}) \circ k \circ (g \circ g^{-1}) = k$$

$$\psi = \varphi^{-1}$$

Existuje inverzné zobrazenie k φ, teda φ je bijekcia.